Constituent quark model for baryons with strong quark-pair correlations and non-leptonic weak transitions of hyperon

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We study the roles of quark-pair correlations for baryon properties, in particular on non-leptonic weak decay of hyperons. We construct the quark wave function of baryons by solving the three body problem explicitly with confinement force and the short range attraction for a pair of quarks with their total spin being 0. We show that the existence of the strong quark-quark correlations enhances the non-leptonic transition amplitudes which is consistent with the data, while the baryon masses and radii are kept to the experiment.

1. Introduction

Properties of light baryons have been extensively studied based on the constituent quark picture, in which the constituent quarks are assumed to be identified with quasi-particles of QCD vacuum. Their results including applications for the two nucleon system are quite consistent with experiments. Despite the success of this approach, understandings of the non-leptonic weak hyperon decay and its $\Delta I = 1/2$ rule are still incomplete[1,2].

It is well known that the factorization as well as the penguin contributions are too small to reproduce the $\Delta I = 1/2$ decays of hyperons. Making use of the soft pion theorem, one can derive the baryon pole contribution to the non-leptonic decay, where initial hyperon B_i changes to the intermediate state baryon B_n by the weak interaction and then B_n emits the pion to produce the final state B_f(and vice versa). The pole approximation with SU(6) spin-flavor symmetry for SU(3) baryons can reproduce relative magnitudes of various hyperon decays very well. However, if one calculates these transition amplitudes using the constituent quark model, the absolute value of the amplitudes is about a half of the experimental data at most[2].

Let us write the effective weak interaction Hamiltonian[3];

$$\mathcal{H}_W = \frac{G_F \sin\theta \cos\theta}{\sqrt{2}} \sum_i c_i(\mu^2)\, O_i + \text{h.c.} \tag{1}$$

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where

$$O_1 = [\bar{u}\gamma_\mu(1-\gamma_5)s][\bar{d}\gamma_\mu(1-\gamma_5)u] \tag{2}$$
$$O_2 = [\bar{d}\gamma_\mu(1-\gamma_5)s][\bar{u}\gamma_\mu(1-\gamma_5)u]\ . \tag{3}$$

Here, we write dominant operators only. Within the baryon pole approximation, the parity-conserving transition amplitude $B_i \to B_f + \pi^a$ is given by,

$$B^a_{fi} = \sum_n \frac{M_i + M_f}{f_\pi}\left(\frac{G^a_{fn}W_{ni}}{M_i - M_n} + \frac{W_{fn}G^a_{ni}}{M_f - M_n}\right)\ , \tag{4}$$

where matrix elements are defined as,

$$\langle B_f|A^{a\mu}|B_n\rangle = G^a_{fn}\bar{u}(f)\gamma^\mu\gamma^5 u(n) \tag{5}$$
$$\langle B_n|\mathcal{H}_W|B_i\rangle = h_{ni}\bar{u}(n)u(i) \tag{6}$$

Axial-vector coupling constants G_{fn} are rather well known quantities. Hence, we face to determine the expectation values of the weak Hamiltonian h_{ni} by the quark model. We recall that the quark models such as Isgur-Karl HO model or MIT bag model give much smaller values than the experimental data[2,4]. It is instructive to rewrite the operator (2,3) in the non-relativistic limit as

$$O_1,\, O_2 \to a^\dagger_d a^\dagger_u (1 - \vec{\sigma}_u\cdot\vec{\sigma}_s)a_u a_s \tag{7}$$

where a_i, $a^\dagger_i$ are annihilation and creation operators of quarks with flavor i. Presence of the spin-projection operator $(1 - \vec{\sigma}_u\cdot\vec{\sigma}_s)$ tells us that only the us pair with their total spin being 0 can contribute to the weak decay process, namely, the weak transition is generated by the two body process between spin-0 quark pairs; $(us)^0 \to (ud)^0$. Now it is clear that this decay amplitude is very sensitive to the correlation of the spin-0 quark pair in the baryons[5]. The standard constituent quark model never incorporates such a correlation properly. However, in fact, more fundamental studies on non-perturbative QCD, e.g. instanton liquid model[6], suggest that there exists the strong attractive correlation for the quark-quark pair with $s = 0$. These considerations naturally lead us to study the quark structure of baryons by taking into account the attractive correlation which could enhance the weak decay amplitudes.

2. Constituent quark model for baryons with spin dependent correlation

Our purpose here is to construct the simplified quark model to deal with the quark pair correlations and thus account for the non-leptonic weak decay. We phenomenologically introduce the effective Hamiltonian which includes the confinement force and the spin-dependent part as;

$$\mathcal{H} = \sum_i \frac{p_i^2}{2m_i} + V_C + V_S + V_0 \tag{8}$$
$$V_C = \sum_{i<j}\frac{1}{2}K\left(\vec{r}_i - \vec{r}_j\right)^2 \tag{9}$$
$$V_S = \begin{cases} 0 & (s=1\ \text{pair}) \\ \sum_{i<j}\frac{C_{SS}}{m_i m_j}\mathrm{Exp}\left[-\left(\vec{r}_i - \vec{r}_j\right)^2/\beta^2\right] & (s=0\ \text{pair}) \end{cases} \tag{10}$$

where m_i are the constituent quark masses, and K, C_{SS}, β are the model parameters. V_0 is the constant parameter which contributes to the over all shift of the resulting spectrum and is chosen to adjust the energy of the lowest state to the nucleon mass. Constituent quark masses are taken to be $m_u = m_d = 330$MeV and $m_s = 500$MeV.

Using this Hamiltonian, we shall solve non-relativistic three body problem rigorously. We use the coupled-rearrangement-channel variational method with infinitesimally-shifted-Gaussian-Lobe basis functions which is developed by ones of the authors[7]. We assume only the isospin symmetry between up and down quarks. The quark wave functions are constructed by the antisymmetrization without any further approximations or assumptions. We note that the SU(6) spin-flavor symmetry should be broken within our formalism because of the spin-dependent correlation. It is interesting to clarify the effects of the broken SU(6) on the non-leptonic weak decay and other baryon properties.

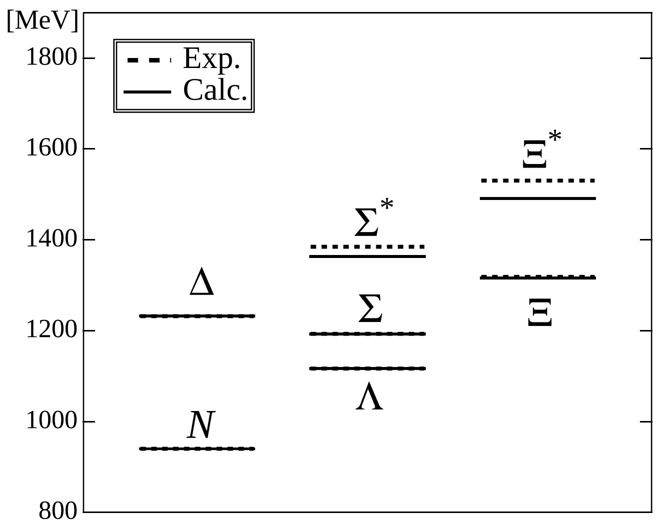

Figure 1. SU(3) baryon mass spectrum: Calculations are shown by the solid lines, and experiments by the dashed ones.

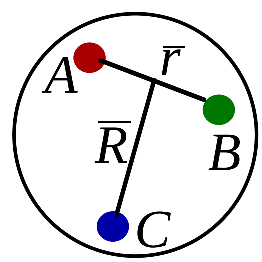

Figure 2. Configuration of three quarks in nucleon

3. Results and discussions

We shall fix the model parameters so as to reproduce the nucleon and Δ masses, proton radius and strength of the non-leptonic hyperon decay. The experimentally measured proton radius includes contributions from both valence quark core part and its meson clouds. It is reasonable to subtract the vector meson dominance contribution $6/m_\rho^2$ from the experimental data of the proton electric radius $(0.86\text{fm})^2$ to obtain the valence quark core radius $\langle r^2 \rangle_{core}$. From this analysis, we determine $\langle r^2 \rangle_{core} = (0.6\text{fm})^2$. We obtain $K = 0.005\text{GeV}^3$, $\beta = 0.5$fm and $C_{SS}/m_u^2 = 1.4$GeV.

We show first in Fig.1 SU(3) baryon mass spectrum. The agreement with the data encourages us to proceed our approach. In order to clarify the effects of the correlation on the nucleon structure, we calculate average distances of the Jacobi coordinate $\bar{r}$ and $\bar{R}$ defined in Fig.2. We find $\bar{r} = 0.92$fm and $\bar{R} = 0.97$fm when the total spin s of the quark

pair A and B is zero, while $\bar{r} = 1.1$fm and $\bar{R} = 0.81$fm in the $s = 1$ case. Apparently, the quark correlation modifies quark distribution in the nucleon.

The weak matrix elements are shown in Table 1. In the left column we show the matrix elements with the quark correlation and the ones without the correlations in the right column. In the absence of the correlation $C_{SS} = 0$, a ratio $\langle p|H_W|\Sigma^+\rangle / \langle n|H_W|\Lambda\rangle = -2.45$ shows a perfect agreement with the SU(6) expectation $\sqrt{6} \simeq -2.4494\cdots$. In the realistic case with the spin-dependent force, one can see the substantial enhancement of the matrix elements and the SU(6) breaking effects. The non-leptonic weak transition amplitudes are tabulated in table 2. We shown the pole contributions only in the second column and additional factorization and penguin contributions are in the third column. Then, we show the total decay amplitudes in the forth column to be compared with the experiments. We find a good agreement for $\Sigma \to N\pi$ decays. The pole contribution for $\Lambda \to n\pi^0$ is small and thus the total amplitude becomes about a half of the data. This is because the SU(6) breaking effects on our quark wave function tends to reduce the matrix element $\langle n|H_W|\Lambda\rangle$ relatively. Improvement of this difficulty is now in progress.

Table 1 Matrix elements of the weak Hamiltonian

	with V_S	without V_S
$\langle n\|\mathcal{H}_W\|\Lambda\rangle$	-0.546	-0.179
$\langle p\|\mathcal{H}_W\|\Sigma^+\rangle$	1.65	0.440

Table 2 P-wave non-leptonic weak transition amplitude

Decay	Pole	others	total	Exp.
$\Sigma \to p\pi^0$	26.0	2.05	28.05	26.24 ± 1.32
$\Sigma^+ \to n\pi^+$	43.4	0.0	43.3	41.83 ± 0.17
$\Lambda \to n\pi^0$	-2.30	-5.02	-7.32	-15.61 ± 1.40

In conclusion, we have studied the roles of the spin-dependent correlations in the baryons for the non-leptonic weak transitions of hyperons. We have constructed the simple non-relativistic quark model with the attractive correlation for the spin-0 quark pair, and solved the three body problem rigorously. Our model reproduces the non-leptonic hyperon decays as well as the baryon masses and radius due to the existence of the spin-0 quark pair correlations.

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